

# A Region Based Active Contour Approach for Liver CT Image Analysis Based on Fuzzy Energy Minimization

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**Abstract**— In this paper a fuzzy energy based active contour image processing technique is proposed and used for the segmentation of liver tumours. This model can detect the tumour boundaries based on fuzzy energy. In classical active contour method the stopping term depends on the gradient of the image. In this model the stopping term depends upon the image colour and spatial segments. This approach converges the tumour boundary very fast, since it calculates the fuzzy energy alterations directly. Various experiments demonstrate that this method is better and more robust than classical active contour methods based on the gradient or other kind of energies.

**Index Terms**— Active contour, Mumford-Shah model, Pseudo level set, Segmentation, Tumour detection, CT image, Fuzzy Energy

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## 1 INTRODUCTION

THIS paper introduces a technique for segmentation of liver tumors. The motivation for this work is, now a day's exact segmentation as well as classification of liver tumors from CT images is quite difficult for pathological analysis of benign and malignancy conditions. When the science behind a disease is poorly understood, experience becomes critical for deciding on a treatment. Certain information regarding to identify the malignancy criteria is wrong in the view of scientific analysis. This is where computer models developed from case studies can be crucial benefit to the physician. This paper introduces a method for aiding in treatment decisions that is based on Active contour based on Fuzzy Energy. The method is applied to a difficult medical diagnosis problem that contains many of the typical difficulties encountered in developing decision aids for patient treatment.

Active contours proposed by Kass et al [1] have been applied to a variety of segmentation problems in medical image analysis especially in liver CT images. Also it will be used in the field of feature extraction, image registration etc. Active contour model in a level set formulation, which is originally introduced by Oscher and Sethian [2]. The most important one is implicit active contour model. Implicit active contour models can be categorized into two categories: one is PDE method [3-7] whose evolution equation is directly constructed; another is the variational level set method [8-10] whose evolution equation is derived from the minimization problem for the energy

functional defined on the level set function. Dual Snakes proposed by Gunn et al [11, 12], dual band active contour [8-10] and also related the same works [13, 14] restrict their search spaces exploiting normal lengths on the initial contour. Active contours based on graph-cut theory [15] is based on the optimization technique. All these models are edge based models. In region based models [16, 17] utilize image statistics inside and outside the contour. Region based active contour model proposed by Chan and Vese [18] based on Mumford-Shah functional, can handle objects with boundaries not defined by gradient. In this case highly constrained models are assumed for pixel intensities within each region. High computational cost is the major drawback of [19-21], overcome by Gibou et al. [22] as well as other researchers. Some of the researchers utilize the importance of k-means algorithm and others solving the PDE based equations with energy functionals. These models are very sensitive to noise as well as ill defined boundaries.

This paper focus on the above mentioned problems. Fuzzy based active contour approach presents the objects deals with the boundaries are not necessarily defined by gradient as well as not very smooth. Discontinuous boundaries are also taken into account for analysis. Fuzzy methods are more accurate and robust in data clustering. Proper clustering have high significance in medical image analysis. Fuzzy based energy minimization principle is used for minimizing the active contour functional. The fuzzy value of energy rejects the weak value of local minimum value. We formulate the model in terms of Pseudo level set functions instead of using Euler-Lagrange equations in classical level set functions. We apply a direct method to solve the partial differential equation without numerical stability constraints. This method improves the computational speed because of without solving Euler-Lagrange equations. The fast solution of PDE based on fuzzy energy provides a stable contour with a desirable resistance to noise.

The paper is organized as follows.

Section II discusses the Mumford Shah model Section III dis-

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cusses the fuzzy energy active contour model and the fuzzy energy concepts. In section IV, we describe briefly Pseudo level set formulation. Section V presents the numerical approximation and algorithm. Section VI presents the experimental results and conclusion are drawn in Section VII.

### II. MUMFORD-SHAH MODEL

Mumford-Shah variational methods have been widely studied in image processing because of their numerical advantages and flexibility. Let  $C$  be the evolving curve and the boundary in a region  $\Omega$ , and  $\omega$  be the open subset of  $\Omega$ . i.e.  $\omega \subset \Omega$  and  $C = \partial\omega$ . Also, inside  $C$  denotes region  $\omega$  and outside  $C$  denotes  $\Omega \setminus \omega$ . If an image  $u_0(x,y)$  is formed by two regions of approximately piecewise-constant intensities, considering the fitting term,

$$F_1(C) + F_2(C) = \int_{inside(C)} |u_0(x,y) - c_1|^2 dx dy + \int_{outside(C)} |u_0(x,y) - c_2|^2 dx dy \quad (1)$$

where  $c_1$  and  $c_2$  the constants depending on  $C$  are the averages of  $u_0$  inside  $C$ .  $C_0$ , the boundary of the object is the minimizer of the fitting term.

### III. FUZZY ENERGY BASED ACTIVE CONTOUR MODEL

In this model we combine the fuzzy energy [23] using an active contour model whose value varies with the local contrast of the image. This combination enables the proposed algorithm to address intensity inhomogeneity and to improve the accuracy of segmentation and its robustness to initialization. Besides, the proposed algorithm incorporates neighborhood spatial information into the membership function to reduce the impact of noise. In this model a membership function is included in the fitting term. It indicates the degree of membership of  $u_0(x,y)$  to the inside of  $C$ . The fitting term is defined as

$$F(C, c_1, c_2, u) = \mu \text{length}(C) + \int_{\Omega} |v(x,y)|^m |u_0(x,y) - c_1|^2 dx dy + \int_{\Omega} |1-v(x,y)|^m |u_0(x,y) - c_2|^2 dx dy \quad (2)$$

The membership function  $v(x,y) \in [0,1]$  is the degree of membership of  $u_0(x,y)$  to the inside of  $C$  and  $m$  is a weighting exponent on each fuzzy membership. In this case, it is obvious that the boundary of the object  $C_0$ , is the minimizer of the fitting term

$$F_1(C_0) + F_2(C_0) \approx 0 \quad (3)$$

The four cases are illustrated in Fig. 1. If the curve  $C$  is outside the object, then  $F_1(C) > 0$  and  $F_2(C) \approx 0$ . If the curve  $C$  is inside the object then  $F_1(C) \approx 0$  and  $F_2(C) > 0$ . If the curve  $C$  is both inside and outside the object then  $F_1(C) > 0$  and  $F_2(C) > 0$ . And of  $C=C_0$ , the energy is minimized and the curve  $C$  is now the boundary. Fig.1 indicates the curve fitting pictorial representation of the mathematical expressions.

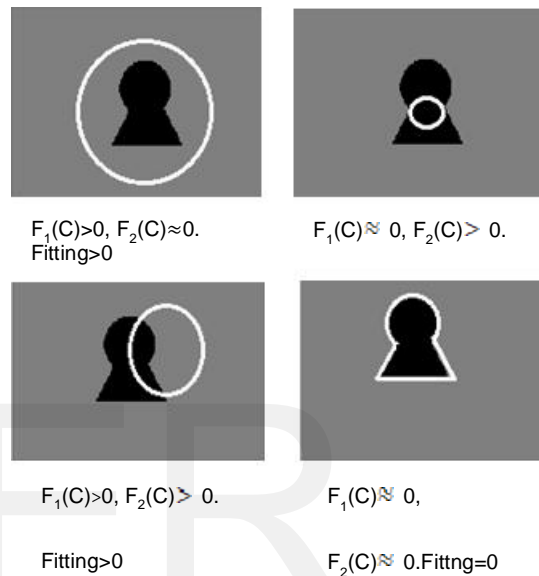


Fig.1. Curve fitting representation

The proposed active contour is based on the minimization of the above fitting term, taking into account the length term of the model  $C$  and the area of the region inside  $C$ . Then the energy functional is defined as

$$F(C, c_1, c_2, u) = \mu \text{length}(C) + \lambda_1 \int_{\Omega} |v(x,y)|^m |u_0(x,y) - c_1|^2 dx dy + \lambda_2 \int_{\Omega} |1-v(x,y)|^m |u_0(x,y) - c_2|^2 dx dy \quad (4)$$

where  $\mu \geq 0, v \geq 0, \lambda_1 > 0$  and  $\lambda_2 > 0$ . The curve  $C_0$  that minimizes  $F$  is the solution to the segmentation problem is given by

$$F(C_0, c_1, c_2, v) = \inf_C F(C, c_1, c_2, v) \quad (5)$$

#### IV.Pseudo Level-Set Formulation

Pseudo level set formulation is defined based on the membership values  $v(x,y)$ .The evolving curve  $C$  is said to be a proper subset of  $u_0(x,y)$  .The pseudo level set of Lipchitz similar function  $u:u_0 \rightarrow \mathbb{R}$  such that

$$\begin{cases} C & = \{(x,y) \in u_0 : v(x,y)=0.5\} \\ \text{inside}(C) & = \{(x,y) \in u_0 : v(x,y)=0.5\} \\ \text{outside}(C) & = \{(x,y) \in u_0 : v(x,y)=0.5\} \end{cases} \quad (6)$$

In medical image analysis ,liver CT images the intensities of pixels belonging to changed and unchanged regions generally have overlapping region. In this case fuzzy techniques which introduce the idea of partial membership by using membership degrees are more appropriate and realistic for describing uncertainties in liver images than hard techniques. The membership degree which reflects the degree of a pixel belonging to a certain region is computed as follows:

$$v(x,y) = \frac{1}{1 + \left( \frac{\lambda_1(u_0(x,y) - c_1)^2}{\lambda_2(u_0(x,y) - c_2)^2} \right)^{\frac{1}{m-1}}} \quad (7)$$

$$c_1 = \frac{\int_{\Omega} [v(x,y)]^m u_0(x,y) dx dy}{\int_{\Omega} [v(x,y)]^m dx dy} \quad (8)$$

$$c_2 = \frac{\int_{\Omega} [1 - v(x,y)]^m u_0(x,y) dx dy}{\int_{\Omega} [1 - v(x,y)]^m dx dy} \quad (9)$$

According to (7), membership degrees are calculated depending on the relative distance between a pixel and the model , as well as on the fuzzy coefficient. The membership degree  $v(x,y)$  from(7) is a precise numeric value. This poses a dilemma of excessive precision in describing uncertain phenomenon. Thus we prefer to view membership degrees as uncertain values rather than, like in traditional fuzzy sets, as certain values. The model taking advantage of fuzzy confidence and based on assumption that the image information related to the global properties of the image contents inside and outside the contour respectively, constructs a new image energy function and could gets global optimal segmentation results.

#### V.Numerical Approximation and Algorithm

In Eqn.4 , the two fitting terms are easy to be computed .The length term is approximated by using Heaviside function and membership function. Note that in our approximation we do not need to differentiate the function in Eqn.4, which would have necessitated in Euler-Lagrange equation of (4).We select the length term given by

$$\sum_{i,j} \sqrt{(Q_{i+1,j} - Q_{i,j})^2 + (Q_{i,j+1} - Q_{i,j})^2} \quad (8)$$

where  $Q_{ij} = H(v(i,j) - 0.5)$ ,  $u(i,j)$  is the value of  $u$  at the  $(i,j)$  pixel and  $H(\cdot)$  is the Heaviside function. The algorithm for the model is as follows. If we do no consider the length term the model converges in less than twenty iterations. Also the parameters  $\lambda_1$  and  $\lambda_2$  are set to be one.

Step 1: Select the tumor portion of the CT image ,  $v > 0.5$  for one part and  $v < 0.5$  for the other.

Step 2: Compute  $c_1$  and  $c_2$  using Eqns. (8) and (9) respectively.

Step 3: Assume the value of the current pixel is  $I_0$  and  $v_0$  its corresponding degree of membership. Calculate the new degree of membership  $v_n$  using Eqn.6 for the pixel  $I_0$  and then compute the difference between the new and old energy using

$$\Delta F = \lambda_1 s_1 \frac{[v_n^m - v_0^m](I_0 - c_1)^2}{s_1 + v_n^m - v_0^m} + \lambda_2 s_2 \frac{[(1 - v_n)^m - (1 - v_0)^m](I_0 - c_2)^2}{s_2 + (1 - v_n)^m - (1 - v_0)^m}$$

where  $s_1 = \sum [v(i,j)]^m$  and  $s_2 = \sum [1 - v(i,j)]^m$  If  $\Delta F < 0$  then change  $v_{ij}$  with  $v_n$  value otherwise, keep the old value.

Step 4: Repeat step 3 for computing the total energy  $F$  of the image.

Step 5: Repeat steps 2 to 4 until the total energy  $F$  remains unchanged.

#### VI. Results and Discussion

In this section we demonstrate the performance of the presented model on various liver tumor CT images. Active contour evolution in the image domain is shown in different images. Also shown the piecewise constant approximation of the image domain given by constants  $c_1$  and  $c_2$ .In our experiments values are selected for  $\lambda_1$  and  $\lambda_2$ . is set to be one. The length parameter  $\mu$  is not same in all experiments, If we have multiple tumors and of any size , then  $\mu$  should be small. If we have to detect large size of tumors and not to detect small size, then  $\mu$  has to be larger.

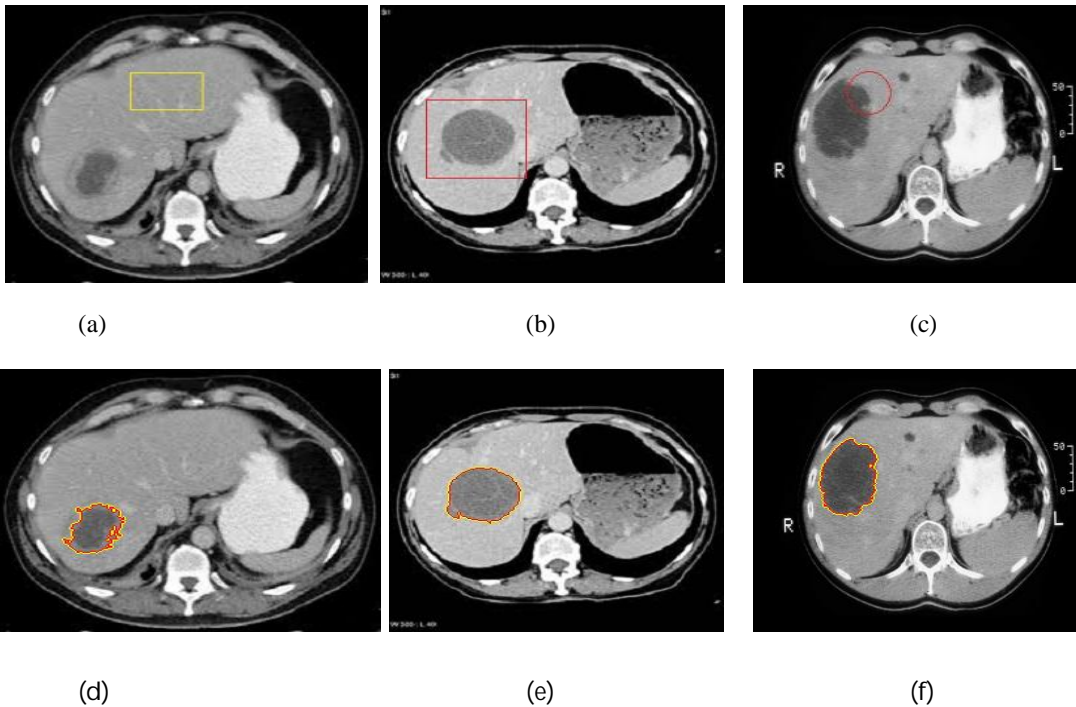


Fig.1. Segmentation of a two phase liver CT image: (a), (b), (c) are three different initial conditions: (d), (e), (f) the result segmented image of (a), (b), (c) respectively. The length parameter was omitted ( $\mu=0$ ).

First the segmentation results on a two phase images is shown in (Fig.1). The length term is omitted ( $\mu=0$ ) since there is no noise. Three different images are taken and different initial conditions are chosen. All of them converge to the correct solution. The interior and exterior contour was automatically detected without considering the second initial model. The algorithm is completely independent to the model's initial position when the length term is omitted. When the length term is included, then the algorithm is dependent to the initial condition. If the length term is increased the model tends to behave

as a rigid one. The length term localizes the model is a property that enforces the model to have a resistance to image noise. Fig.2 shows the evolution of the contour in noisy CT liver image. For analysis an interior contour is selected. Contours are automatically detected without considering a second initial curve. Jacobi iteration is used for energy minimization. The nature of the model and the iteration process allows the automatical change of the topology.

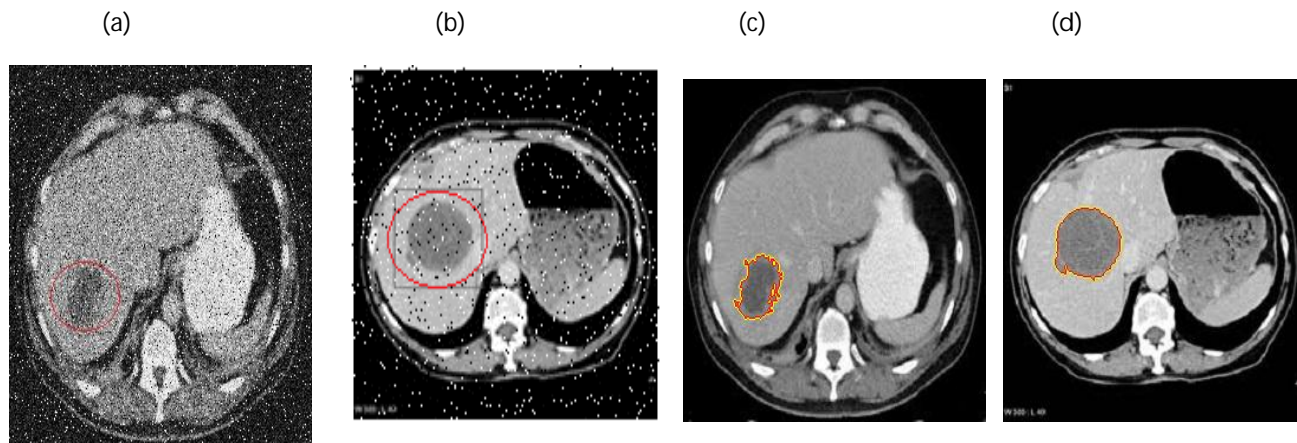


Fig.2. Detection of tumor from noisy images with various shapes and interior contour(Salt and Pepper noise).(a) and (b) are two noisy images: (c) and (d) the segmented images of (a) and (b) respectively. The length parameter is set equal to  $\mu=0.25$ .

Fig.3 demonstrates the model can detect with low intensity and blurred boundaries. The interior contour of the tumor is automatical-

ly detected. In Fig.4 and Fig.5 images with multiple tumors with different intensity are used. The selection of the length parameter is different for fig.4 and fig.5. Fig.7 shows an example where other energy based active contour algorithm fail to detect the tumor lying in an endless loop. The condition for this method is  $u > 0.5$  on the tumor region and  $u < 0.5$  on the background region. Consider the two regions have the same intensities. The algorithm converges after 80 iteration to the object boundary using Jacobi iteration. The length parameter is assumed to be zero in this experiment. Fig.8, Fig.9 and Fig.10 demonstrates how the proposed algorithm could detect the tumor regions for different values of iteration and the length parameter was set to  $\mu = 0.60$ ,  $\mu = 0.70$  and  $\mu = 0.80$  respectively. Contour plots are also shown along with the segmented results.

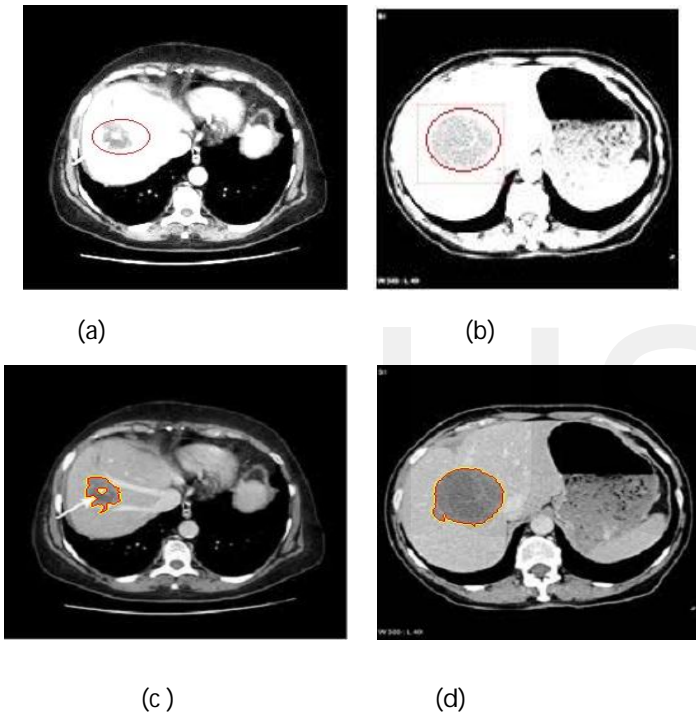


Fig.3. Detection of blurred tumors and low intensity. (a) and (b) are two blurred tumor images; (c) and (d) the segmented images of (a) and (b) respectively. The length parameter is set equal to  $\mu = 0.065$ .

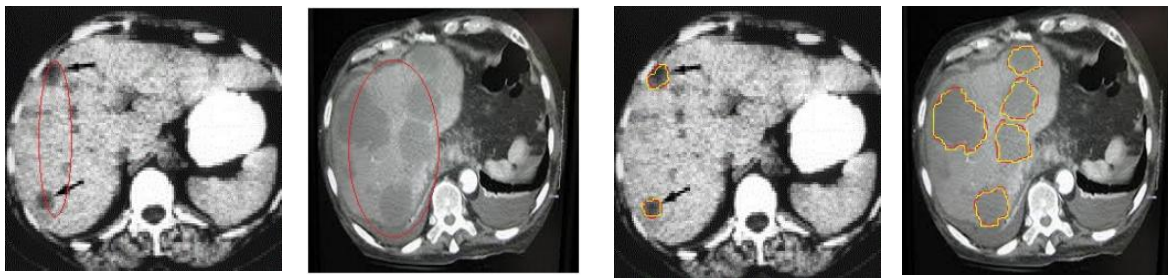


Fig.5. Detection of geometrically similar objects with chromatic identity. (a) and (b) are two CT tumor images; (c) and (d) the segmented images of (a) and (b) respectively. The length parameter is set equal to  $\mu = 0.8$ .

Fig.11 shows the comparison of Chan-Vese method, Gibou-Fedkiw method and the presented method. First column shows the CT liver images with noisy(both uniform and Gaussian) background and blurred images, Due to the length parameter the presented model has a remarkable resistance to noise converging almost clear tumors. Second column depicts how our model detects tumors on noisy or blurred images containing tumors in (first column). The third column of Fig.10 shows the results of objects detection exploiting the Chan-Vese method, while the last column depicts the results of the Gibou-Fedkiw method.

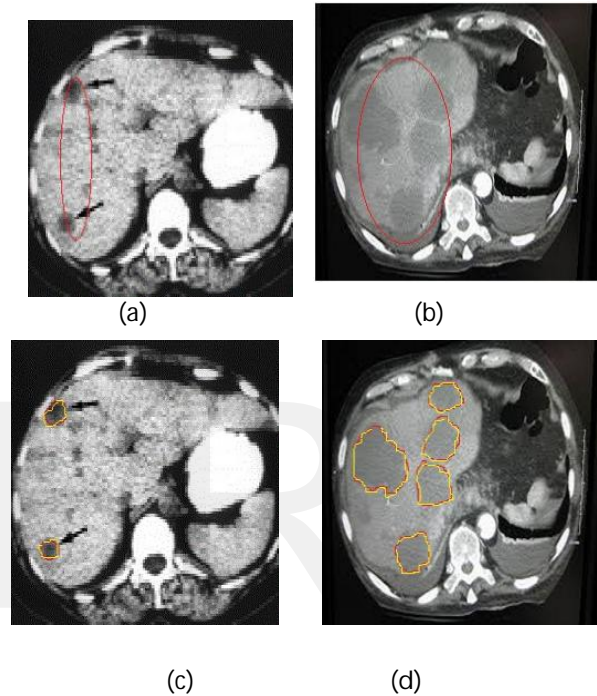


Fig.4. Detection of geometrically similar objects with different intensities. (a) and (b) are two CT tumor images; (c) and (d) the segmented images of (a) and (b) respectively. The length parameter is set equal to  $\mu = 0.008$ .

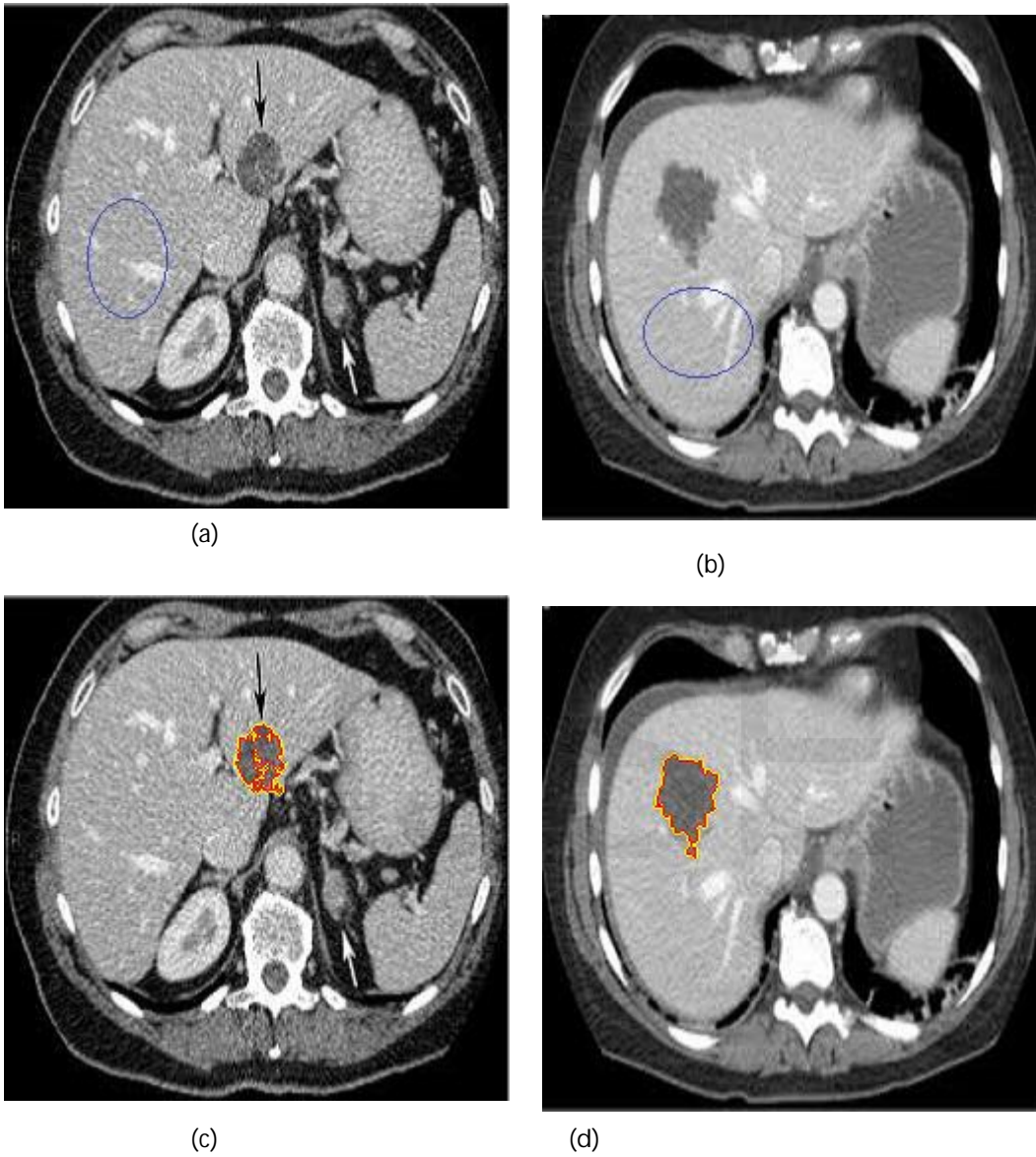


Fig.6. Detection liver tumor with very smooth contour. (a) and (b) are two CT tumor images: (c) and (d) the segmented images of (a) and (b) respectively. The length parameter is set equal to  $\mu=0.003$

## VII. Conclusion

In this paper, a fast and robust model for active contours to detect the tumors in CT liver images is introduced. This method makes use of the global statistical information as well as the local information, which makes our method robust to noise. The model can detect tumors whose boundaries are not necessarily defined by gradient, due to the fact that it is based on an energy minimization algorithm, and not on an edge function as the most classical active contour models. This energy minimization technique is based on fuzzy logic and is used as model to detect the tumor boundary. In most of the classical active contours the stopping term of the model evolution depend on the gradient of the image. In this method evolution depends on the spatial segments of the image. The fuzziness of the energy provides a balanced technique with a strong ability to reject weak local minima. Also, it is not needed to smooth the initial images, even if they are very noisy, since the model very well detect and preserve the locations of the boundaries. The initial position of the model can be any where in the image, and it does not necessarily surrounds the tumor regions to be detected.. This method calculates the fuzzy energy alterations directly without numerical stability constraints. So it converges to the desired tumor boundary very fast.

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